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Numerical Simulation of Shock Wave Phenomena using Smoothed Particle Hydrodynamics

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Abstract. A modified version of the Lagrangian particle method, Smoothed Particle Hydrodynamics (SPH), is tested against typical shock wave benchmark tests. The method allows a stepwise variable smoothing length and a regularization process ensures a quasi regular particle distribution. The results obtained for a Mach 3 windtunnel, an oblique wedge and a shock wave expansion are quite promising.

1 Introduction

The method of Smoothed Particle Hydrodynamics (SPH) was introduced by Lucy (1977) and Gingold & Monaghan (1977) for gas dynamic problems related to astrophysics. A wide range of different subjects has over the years been studied with the method. The problems range from star collisions involving either compact objects or more sun-like stars, studies of two fluid fluctuations in an expanding universe, supernova explosions to magnetic phenomena. Work has also been put into simulating impact and cratering of planetary surfaces due to the collisions with comets and asteroids.

Even if most work is still in the field of astrophysics, the method has now been adopted by other research fields, such as for instance in the material modeling of impacts on different types of surfaces. For a thorough description of the variety of subjects studied, the reader is recommended the review article by Monaghan (1992a).

2 The SPH method

2.1 Fundamentals

The SPH method is a Lagrangian numerical method, in which a collection of interacting discrete simulation particles is used to model a continuous fluid flow. Each particle j is given characteristic properties such as mass m_j, position $\mathbf{r}_j$, velocity $\mathbf{v}_j$, density ρ_j and thermal energy density u_j. The instantaneous properties of the fluid at an arbitrary position $\mathbf{r}$, are derived from the properties of the surrounding simulation particles. The simulation particles in turn must satisfy equations of motion of proper form such that also the temporal properties of the continuous fluid flow are correctly described. Through this procedure the hydrodynamic equations are reduced to a set of ordinary differential equations.

The SPH method is based on interpolation theory, in which an arbitrary function $f(r)$ is approximated as a kernel estimate of itself,

$$\langle f(\mathbf{r})\rangle = \int f(\mathbf{r}')W(\mathbf{r}-\mathbf{r}',h)d\mathbf{r}'. \tag{1}$$

The kernel function $W(\mathbf{r}-\mathbf{r}',h)$ has a characteristic width h, which is called the smoothing length. It is a non-negative function, and integrated over the total domain it should normalize to unity. As $h \to 0$ it should approach a delta function. Thus, in this limit $\langle f(\mathbf{r})\rangle = f(\mathbf{r})$. The transition to a discrete particle formulation is accomplished by identifying $\rho(\mathbf{r}')d\mathbf{r}'$ as the differential mass element dm', and approximating the integral in (1), as a sum over discrete particles, that is

$$\langle f(\mathbf{r})\rangle \approx \sum_j m_j \frac{f_j}{\rho_j} W(\mathbf{r}-\mathbf{r_j},h). \tag{2}$$

Spatial derivatives of field variables are approximated by replacing the kernel function in (2) by its spatial derivative,

$$\langle \nabla f(\mathbf{r})\rangle \approx \sum_j m_j \frac{f_j}{\rho_j} \nabla W(\mathbf{r}-\mathbf{r}_j,h). \tag{3}$$

For this reason the kernel function W must have at least first order continuous derivatives. In addition W is usually chosen to have finite support to limit the number of nearest neighbor particles involved in the summation, and thus also limit smearing effects.

There is no unique method as to how the proper equations of motion for the particle properties $\mathbf{r_j}$, $\mathbf{v_j}$, ρ_j and

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